International Baccalaureate®
Baccalauréat International
Bachillerato Internacional

22137204

MATHEMATICS
HIGHER LEVEL
PAPER 2

Candidate session number

0	0							

Friday 10 May 2013 (morning)

Examination code

2	2	1	3	–	7	2	0	4

2 hours

INSTRUCTIONS TO CANDIDATES

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Section A: answer all questions in the boxes provided.
- Section B: answer all questions in the answer booklet provided. Fill in your session number on the front of the answer booklet, and attach it to this examination paper and your cover sheet using the tag provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the ***Mathematics HL and Further Mathematics SL*** *information booklet* is required for this paper.
- The maximum mark for this examination paper is *[120 marks]*.

14 pages

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. In particular, solutions found from a graphic display calculator should be supported by suitable working, for example, if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

SECTION A

*Answer **all** questions in the boxes provided. Working may be continued below the lines if necessary.*

1. *[Maximum mark: 4]*

The marks obtained by a group of students in a class test are shown below.

Marks	Frequency
5	6
6	k
7	3
8	1
9	2
10	1

Given the mean of the marks is 6.5, find the value of k.

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

2. *[Maximum mark: 5]*

Find the value of k such that the following system of equations does not have a unique solution.

$$\begin{aligned} kx + y + 2z &= 4 \\ -y + 4z &= 5 \\ 3x + 4y + 2z &= 1 \end{aligned}$$

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

3. *[Maximum mark: 5]*

Emily walks to school every day. The length of time this takes can be modelled by a normal distribution with a mean of 11 minutes and a standard deviation of 3 minutes. She is late if her journey takes more than 15 minutes.

(a) Find the probability she is late next Monday. *[2 marks]*

(b) Find the probability she is late at least once during the next week (Monday to Friday). *[3 marks]*

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

4. *[Maximum mark: 6]*

Let $f(x) = \sin(x - 1)$, $0 \le x \le \frac{\pi}{2} + 1$

Find the volume of the solid formed when the region bounded by $y = f(x)$, and the lines $x = 0$, $y = 0$ and $y = 1$ is rotated by 2π about the y-axis.

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

5. *[Maximum mark: 6]*

A rectangle is drawn around a sector of a circle as shown. If the angle of the sector is 1 radian and the area of the sector is $7\,\text{cm}^2$, find the dimensions of the rectangle, giving your answers to the nearest millimetre.

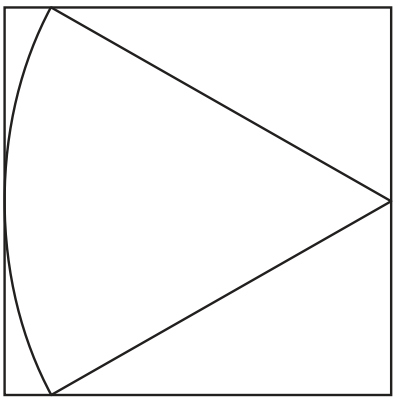


diagram not to scale

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

6. *[Maximum mark: 7]*

A polynomial $p(x)$ with real coefficients is of degree five. The equation $p(x) = 0$ has a complex root $2 + \mathrm{i}$. The graph of $y = p(x)$ has the x-axis as a tangent at $(2, 0)$ and intersects the coordinate axes at $(-1, 0)$ and $(0, 4)$.

Find $p(x)$ in factorised form with real coefficients.

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

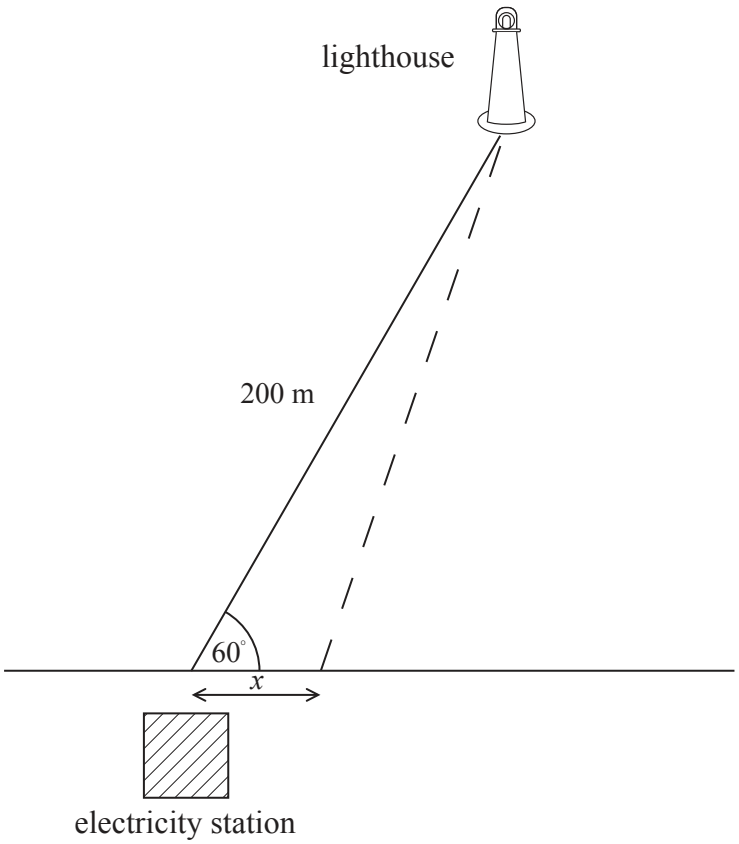
. . . . . .

. . . . . .

. . . . . .

7. *[Maximum mark: 6]*

An electricity station is on the edge of a straight coastline. A lighthouse is located in the sea 200 m from the electricity station. The angle between the coastline and the line joining the lighthouse with the electricity station is 60°. A cable needs to be laid connecting the lighthouse to the electricity station. It is decided to lay the cable in a straight line to the coast and then along the coast to the electricity station. The length of cable laid along the coastline is x metres. This information is illustrated in the diagram below.



(This question continues on the following page)

(Question 7 continued)

The cost of laying the cable along the sea bed is US$80 per metre, and the cost of laying it on land is US$20 per metre.

(a) Find, in terms of x, an expression for the cost of laying the cable. *[4 marks]*

(b) Find the value of x, to the nearest metre, such that this cost is minimized. *[2 marks]*

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

8. *[Maximum mark: 7]*

Three boys and three girls are to sit on a bench for a photograph.

(a) Find the number of ways this can be done if the three girls must sit together. *[3 marks]*

(b) Find the number of ways this can be done if the three girls must all sit apart. *[4 marks]*

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

9. *[Maximum mark: 7]*

(a) Prove that the equation $3x^2 + 2kx + k - 1 = 0$ has two distinct real roots for all values of $k \in \mathbb{R}$. *[4 marks]*

(b) Find the value of k for which the two roots of the equation are closest together. *[3 marks]*

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

10. *[Maximum mark: 7]*

A ferry carries cars across a river. There is a fixed time of T minutes between crossings. The arrival of cars at the crossing can be assumed to follow a Poisson distribution with a mean of one car every four minutes. Let X denote the number of cars that arrive in T minutes.

(a) Find T, to the nearest minute, if $P(X \leq 3) = 0.6$. *[3 marks]*

It is now decided that the time between crossings, T, will be 10 minutes. The ferry can carry a maximum of three cars on each trip.

(b) One day all the cars waiting at 13:00 get on the ferry. Find the probability that all the cars that arrive in the next 20 minutes will get on either the 13:10 or the 13:20 ferry. *[4 marks]*

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

*Do **NOT** write solutions on this page.*

SECTION B

*Answer **all** questions in the answer booklet provided. Please start each question on a new page.*

11. *[Maximum mark: 20]*

Consider the points $\mathrm{P}(-3, -1, 2)$ and $\mathrm{Q}(5, 5, 6)$.

(a) Find a vector equation for the line, L_1, which passes through the points P and Q. *[3 marks]*

The line L_2 has equation

$$\boldsymbol{r} = \begin{pmatrix} -4 \\ 0 \\ 4 \end{pmatrix} + s \begin{pmatrix} 5 \\ 2 \\ 0 \end{pmatrix}.$$

(b) Show that L_1 and L_2 intersect at the point $\mathrm{R}(1, 2, 4)$. *[4 marks]*

(c) Find the acute angle between L_1 and L_2. *[3 marks]*

Let S be a point on L_2 such that $\left|\overrightarrow{\mathrm{RP}}\right| = \left|\overrightarrow{\mathrm{RS}}\right|$.

(d) Show that one of the possible positions for S is $\mathrm{S}_1(-4, 0, 4)$ and find the coordinates of the other possible position, S_2. *[6 marks]*

(e) Find a vector equation of the line which passes through R and bisects $\mathrm{P}\overline{\mathrm{RS}_1}$. *[4 marks]*

*Do **NOT** write solutions on this page.*

12. *[Maximum mark: 21]*

A particle, A, is moving along a straight line. The velocity, v_A ms^{-1}, of A t seconds after its motion begins is given by

$$v_A = t^3 - 5t^2 + 6t .$$

(a) Sketch the graph of $v_A = t^3 - 5t^2 + 6t$ for $t \geq 0$, with v_A on the vertical axis and t on the horizontal. Show on your sketch the local maximum and minimum points, and the intercepts with the t-axis. *[3 marks]*

(b) Write down the times for which the velocity of the particle is increasing. *[2 marks]*

(c) Write down the times for which the magnitude of the velocity of the particle is increasing. *[3 marks]*

At $t = 0$ the particle is at point O on the line.

(d) Find an expression for the particle's displacement, x_A m, from O at time t. *[3 marks]*

A second particle, B, moving along the same line, has position x_B m, velocity v_B ms^{-1} and acceleration, a_B ms^{-2}, where $a_B = -2v_B$ for $t \geq 0$. At $t = 0$, $x_B = 20$ and $v_B = -20$.

(e) Find an expression for v_B in terms of t. *[4 marks]*

(f) Find the value of t when the two particles meet. *[6 marks]*

*Do **NOT** write solutions on this page.*

13. *[Maximum mark: 19]*

The function f has inverse f^{-1} and derivative $f'(x)$ for all $x \in \mathbb{R}$. For all functions with these properties you are given the result that for $a \in \mathbb{R}$ with $b = f(a)$ and $f'(a) \neq 0$

$$(f^{-1})'(b) = \frac{1}{f'(a)}.$$

(a) Verify that this is true for $f(x) = x^3 + 1$ at $x = 2$. *[6 marks]*

(b) Given that $g(x) = xe^{x^2}$, show that $g'(x) > 0$ for all values of x. *[3 marks]*

(c) Using the result given at the start of the question, find the value of the gradient function of $y = g^{-1}(x)$ at $x = 2$. *[4 marks]*

(d) (i) With f and g as defined in parts (a) and (b), solve $g \circ f(x) = 2$.

(ii) Let $h(x) = (g \circ f)^{-1}(x)$. Find $h'(2)$. *[6 marks]*

Please **do not** write on this page.

Answers written on this page
will not be marked.